



ATAR course examination, 2019

Question/Answer booklet

MATHEMATICS SPECIALIST

Section One: Calculator-free

Place one of your candidate identification labels in this box.
Ensure the label is straight and within the lines of this box.

WA student number: In figures

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In words

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional
answer booklets used
(if applicable):

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.



Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	9	9	50	47	35
Section Two: Calculator-assumed	10	10	100	85	65
Total					100

Instructions to candidates

1. The rules for the conduct of the Western Australian external examinations are detailed in the *Year 12 Information Handbook 2019*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section One: Calculator-free**35% (47 Marks)**

This section has **nine** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 50 minutes.

Question 1**(4 marks)**

Using the identity $2\sin A \cos B = \sin(A + B) + \sin(A - B)$, evaluate exactly the definite integral

$$\int_0^{\frac{\pi}{2}} 6\sin\left(\frac{5x}{2}\right)\cos\left(\frac{x}{2}\right) dx.$$

Question 2

(6 marks)

Consider the function $P(z) = z^4 - 2z^3 + 14z^2 - 8z + 40$, defined over the complex numbers.

(a) Show that $(z - 2i)$ is a factor of $P(z)$. (2 marks)

(b) Hence or otherwise, solve the equation $P(z) = 0$, giving solutions in the form $a + bi$. (4 marks)

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Question 3

(5 marks)

- (a) Given that $\frac{2x^2 + 5x + 6}{x^2(x + 3)} = \frac{a}{x} + \frac{b}{x^2} + \frac{c}{x + 3}$, determine the values of a , b and c . (2 marks)

- (b) Hence determine $\int \frac{2x^2 + 5x + 6}{x^2(x + 3)} dx$. (3 marks)

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Question 4

(7 marks)

Functions f , g and h are defined such that:

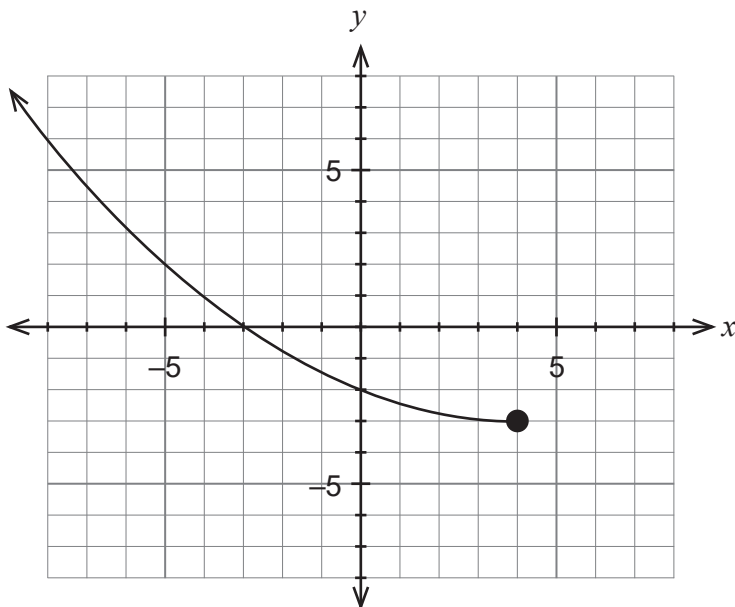
$$f(x) = \frac{1}{x-1}, g(x) = x^2, h(x) = \sqrt{x}.$$

- (a) Determine the defining rule for $f(h(x))$. (1 mark)
- (b) Determine the domain for $f(h(x))$. (2 marks)
- (c) Determine the range for $f(h(x))$. (2 marks)
- (d) Is it true that $f(h(g(x))) = \frac{1}{x-1} = f(x)$? Justify your answer. (2 marks)

Question 5

(6 marks)

The graph of $y = g(x)$ is shown below.



(a) Sketch the graph of $y = g^{-1}(x)$ on the axes above. (3 marks)

(b) Given that $g(x) = \frac{1}{16}(x - 4)^2 - 3$ where $x \leq 4$, determine the defining rule for $y = g^{-1}(x)$. (3 marks)

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Question 6

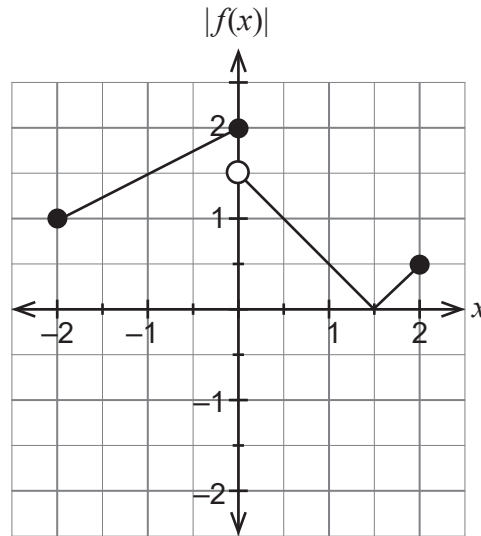
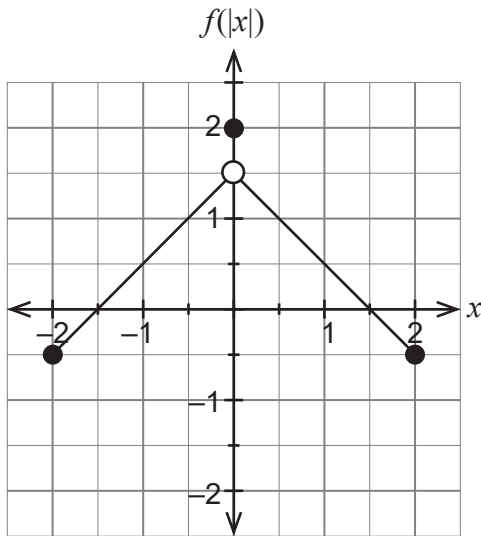
(6 marks)

Using the substitution $x = 2\sin\theta$, evaluate exactly $\int_0^{\sqrt{3}} \sqrt{1 - \frac{x^2}{4}} dx$.

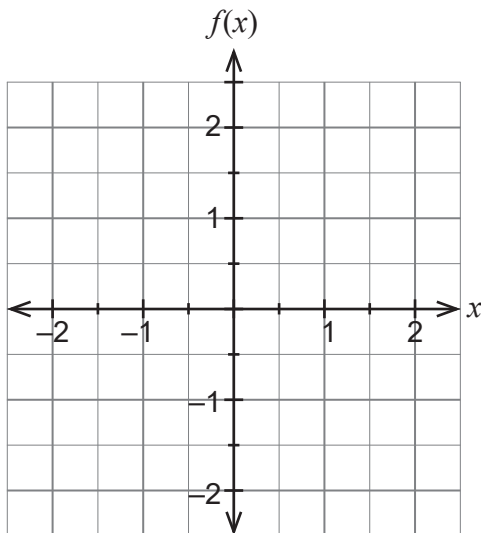
Question 7

(4 marks)

The graphs of $y = f(|x|)$ and $y = |f(x)|$ are shown below.



Given that $y = f^{-1}(x)$ is also a function, sketch a possible graph for $y = f(x)$ on the axes below. Justify your answer considering $y = f^{-1}(x)$.



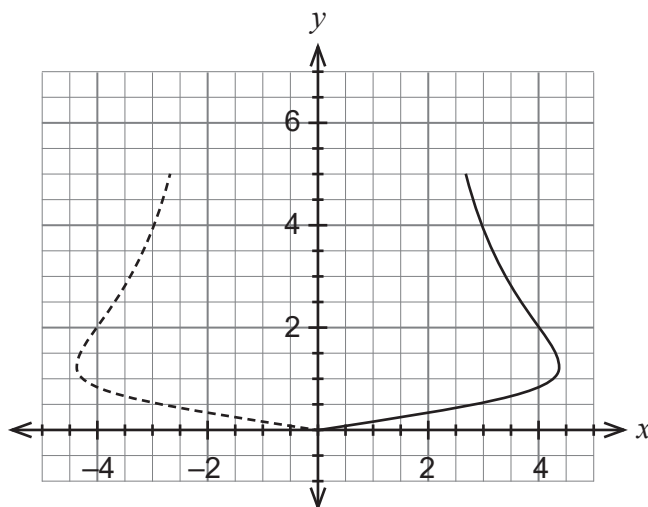
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A spare grid is provided at the end of this Question/Answer booklet. If you need to use it, cross out this attempt and indicate clearly that you have redrawn it on page 15.

Question 8

(5 marks)

The top part of a wine glass is modelled by rotating the graph of $x^2 = y^2(36 - x^2y)$ from $y = 0$ to $y = 5$ about the y axis as shown below. Dimensions are measured in centimetres.



- (a) Show that the volume, $V \text{ cm}^3$, when the glass is full is given by

$$V = \pi \int_0^5 \frac{36y^2}{1 + y^3} dy.$$

(1 mark)

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(b) Determine the exact volume V cm³.

(4 marks)

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Question 9

(4 marks)

Consider the complex equation $z^n - 1 = 0$, where n is any positive integer $n \geq 3$.

If the roots are designated as $z_0, z_1, z_2, \dots, z_{n-1}$, then determine the exact value for the product of the roots $p = z_0 \times z_1 \times z_2 \times \dots \times z_{n-1}$.

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Supplementary page

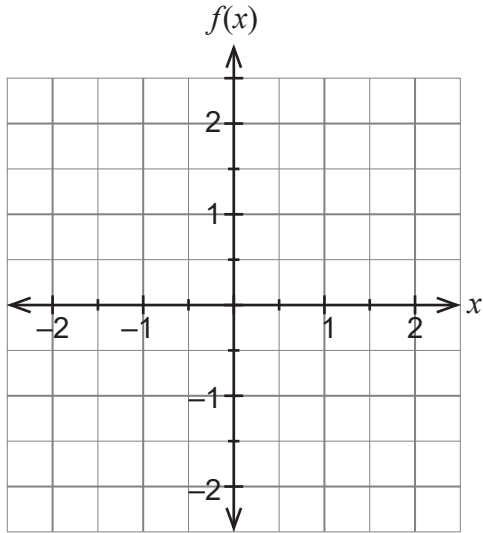
Question number: _____

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Supplementary page

Question number: _____

Spare grid for Question 7.



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